

Homework 2

PHY 414: Introduction to Quantum Computing

Note: Please submit HW before Sunday 10 PM. I would like to upload the solution on Sunday night for you to see before the midterm on Monday. Submit a scanned copy in pdf form contain all the paper work of three problems and a single Notebook for all three problems.

Problem 1: The 'Qiskit' library does not contain a built-in method for implementing the controlled S-gate (as seen in Activity 3). You are required to make the controlled-S gate by using other available gates in the library. This can be done by using only controlled X-gate(s) and Z rotation-gate(s). Your work should be done in Python.

Usage of controlled X-gate

```
yourQuantumCircuit.cx(controlQubit, targetQubit)
```

Usage of Z rotation-gate

```
yourQuantumCircuit.rz(phi, qubit)
```

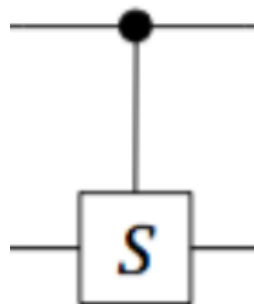


Figure 1: Controlled S-gate

Input	Output
$ 00\rangle$	$ 00\rangle$
$ 01\rangle$	$ 01\rangle$
$ 10\rangle$	$ 10\rangle$
$ 11\rangle$	$i 11\rangle$

Truth table for the controlled S-gate

$$S = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + i|11\rangle\langle 11| \quad (1)$$

Problem 2: In the class, we showed that Deutsch–Josza algorithm states that the function in the phase oracle is constant if the output is state $|0\rangle^{\otimes n}$ and the function is balanced if the output is any other state. This was when the input was initialized to state $|0\rangle^{\otimes n}$. For this problem, assume that the initial state of all qubits is $|1\rangle^{\otimes n}$. Now, prove that the function is constant if the output is $|1\rangle^{\otimes n}$ and is balanced if any other state is output. Check it in qiskit as well.

Problem 3: Implement Bernstein–Vazirani circuit for secret bit string $s = 1011$ and run your code on two quantum computers of IBM for 1000 and 5000 shots (on each of these quantum computers). By comparing your results with the simulator’s results, can you decide which of these quantum computers has less noise?

You should submit your jobs at least two days before the deadline because there can be a long wait time on *yorktown*. Also, submit all four simulations on IBMQ at the same time to save time, instead of submitting one simulation at one time.